Spring 2014 **Math 601** Name: ______________________

Quiz 3 sample

Question 1. (10 pts)

Determine if the given subset is a subspace of the corresponding vector space. (**Show work!**).

(a) (5 pts) The subset of $\mathbb{R}^3$:

$$W = \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z \geq 0\}$$

Solution: Consider $u = (1, 1, 1)$. By assumption, u is a vector in W. Now take $k = -1 \in \mathbb{R}$. We have

$$(-1)u = (-1, -1, -1),$$

but $(-1) + (-1) + (-1) = -3 < 0$. So $(-1)u$ is not in W.
We conclude that W is not a subspace.

(b) (5 pts) Let $\mathcal{M}_{n\times n}$ be the vector space of all real $n \times n$ matrices.

$$W = \{A \in \mathcal{M}_{n\times n} \mid \mathrm{tr}(A) = 0\}$$

Solution: Recall that

$$\mathrm{tr}(0_{n\times n}) = 0,$$

$$\mathrm{tr}(A + B) = \mathrm{tr}(A) + \mathrm{tr}(B),$$

$$\mathrm{tr}(kA) = k\mathrm{tr}(A).$$

Now use these to verify the conditions for W to be subspace.
We conclude that W is a subspace.

Question 2. (10 pts)

Given $u_1 = (1, 0, 1)$, $u_2 = (0, 1, 2)$ and $u_3 = (2, 2, 6)$ in $\mathbb{R}^3$. Determine whether $v = (0, 0, 1)$ is in the span of $\{u_1, u_2, u_3\}$.

Solution: form the matrix B whose columns are u_1, u_2, u_3:

$$B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 6 \end{bmatrix}$$

We consider the augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 2 & 6 & 1 \end{array}\right]$$

Use Gaussian elimination again and we have

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

which implies the system has no solution. So v is not in the span of $\{u_1, u_2, u_3\}$.